2.4 The gravity anomalies of simple shapes

- The sphere
- The horizontal cylinder
- The horizontal truncated thin sheet
- The horizontal cylinder of arbitrary cross section

The gravity anomalies of several bodies of simple shape serve to illustrate the form and size of anomalies expected in typical field surveys.

The sphere

The gravity anomaly of a sphere is that of a point mass at the sphere center equal to the product of the density and volume of the sphere. This model is also the simplest example of the non-unique property of the gravity field: only the ρV product governs the anomaly and neither the size or density can be determined individually.

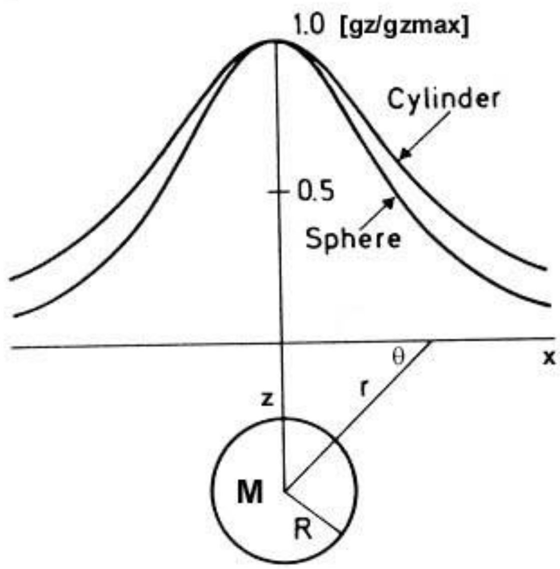
$$g = \frac{GM}{r^2} = \frac{GM}{x^2 + z^2}$$

$$g_z = g \sin\theta = g \frac{z}{(x^2 + z^2)^{1/2}} = GM \frac{z}{(x^2 + z^2)^{3/2}}$$

$$M = \frac{4}{3}\pi R^3 \Delta\rho$$

and so

$$g_z = G \frac{4}{3}\pi R^3 \Delta\rho \frac{z}{(x^2 + z^2)^{3/2}}$$



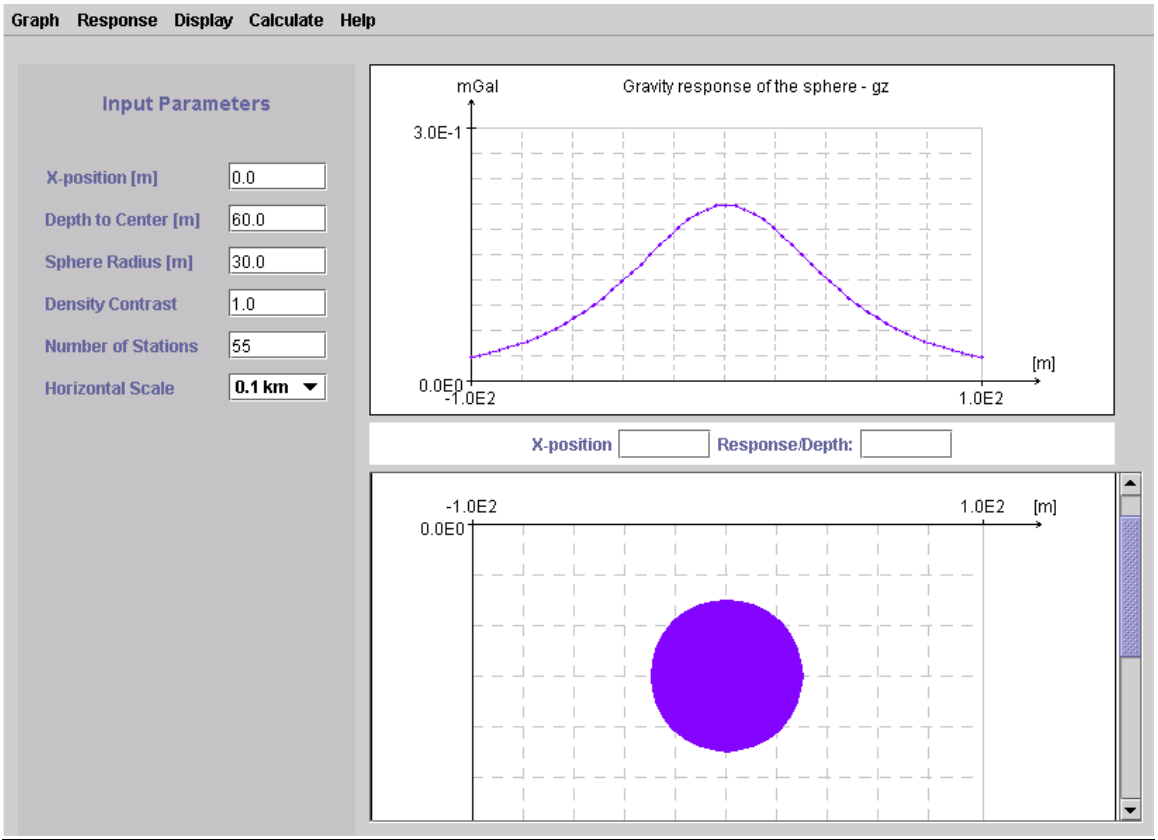
Directly above the sphere the maximum anomaly is:

$$g_{z_{max}} = G\frac{4}{3}\pi\Delta\rho\frac{R^3}{z^2}$$

A useful rule of thumb for the depth of the center of mass can be derived from the half width of the anomaly at its half height:

Half width at half height is equal to 0.77 z, or

$$z = 1.3\ x_{1/2}$$



Response of the Sphere

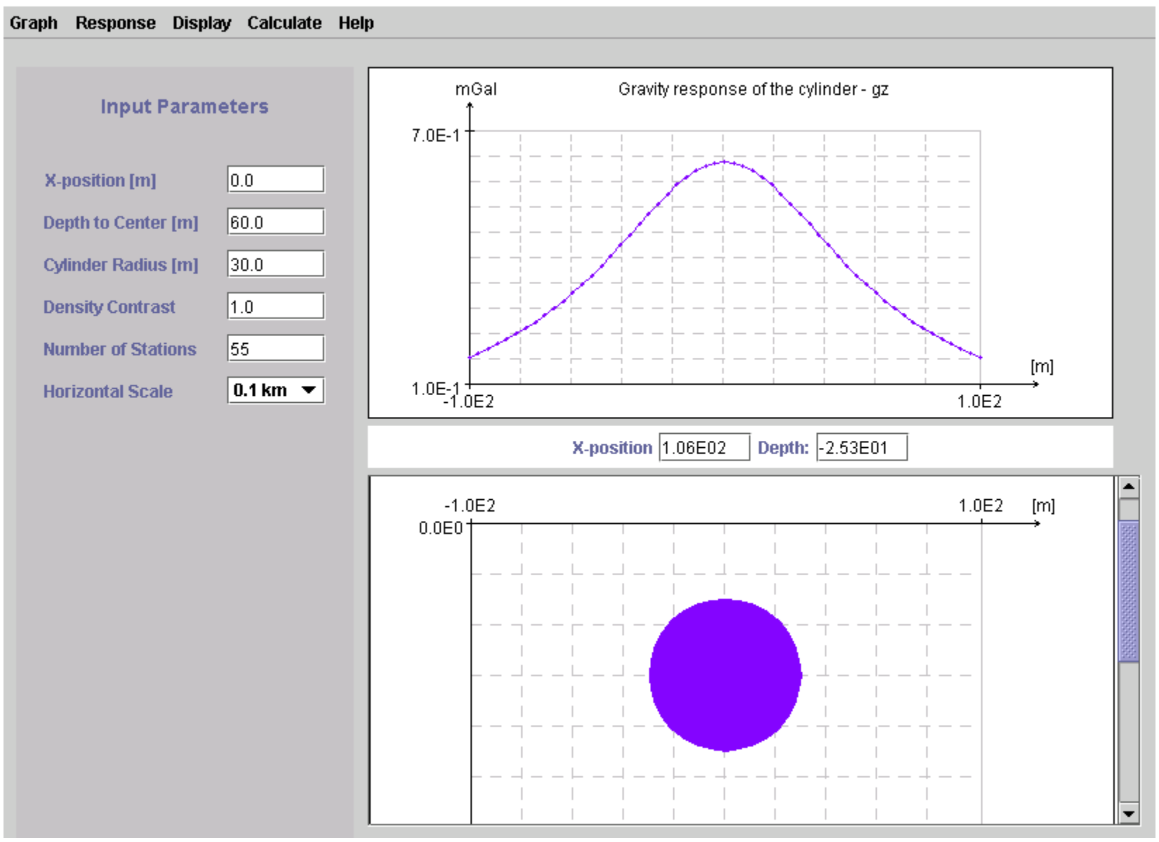
The horizontal cylinder

The anomaly of a horizontal cylinder of the same cross section as the sphere is given by:

$$g_z = G2\pi R^2 \Delta\rho \frac{z}{x^2 + z^2}$$

and $z = x_{1/2}$

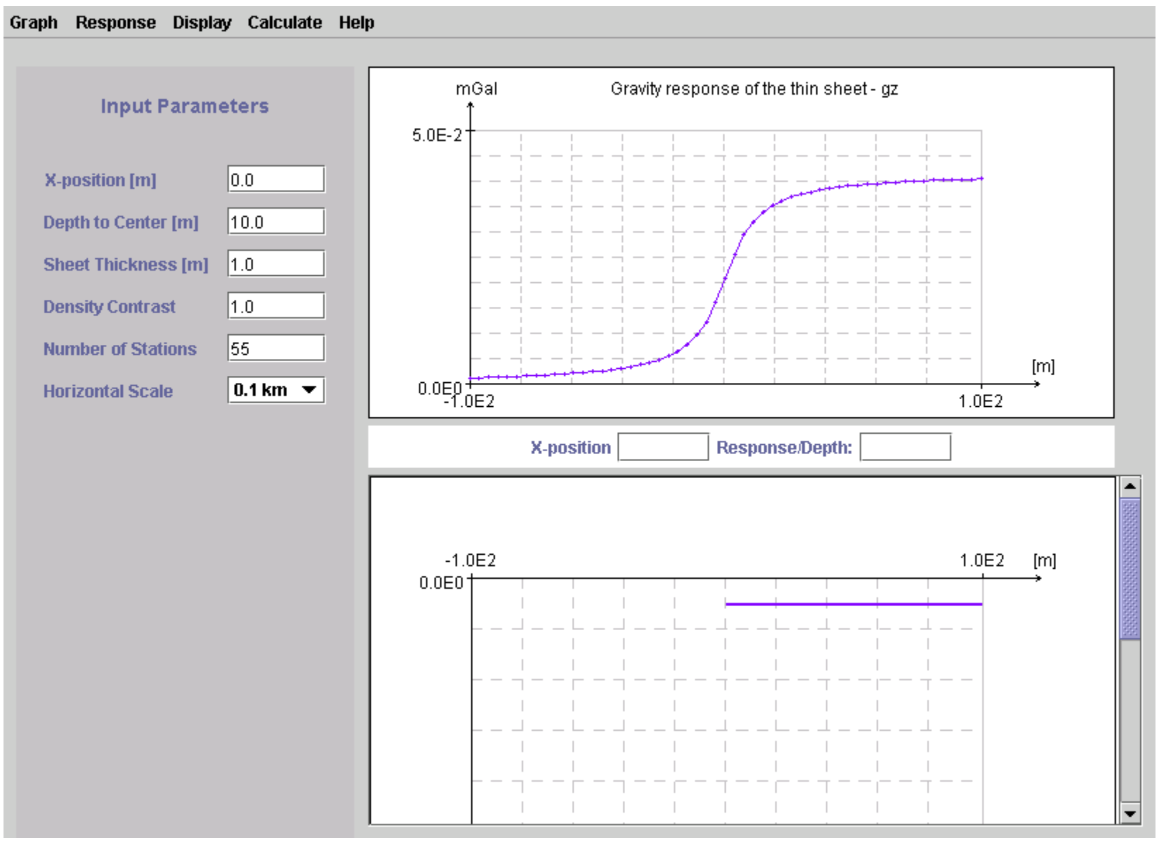
Note that the cylinder anomaly falls off more slowly than the sphere, $1/z$ rather than $1/z^2$, and that the anomaly is broader than that of the sphere. While it might be difficult to distinguish between these two bodies in a survey profile, a map would show the parallel linear contours of the cylinder vs. the circular contours of the sphere.



Response of the Horizontal Cylinder

The horizontal truncated thin sheet

The horizontal truncated thin sheet is a useful approximation for the anomaly of a bedded formation displaced by a fault. An actual layered sequence could be modeled by superposing the contributions of all the layers exhibiting density contrasts. In the figure below only one layer of contrasting density is used.



Response of the Horizontal Thin Sheet

When the depth, h, is greater than the thickness, t, the anomaly in the vertical gravity is:

$$\Delta g_z = 2 G \Delta\rho t \{ \pi/2 + \tan^{-1}(x/h)\}$$

As x goes to infinity $\Delta g_z \rightarrow 2\pi G \Delta\rho t$ which is just the anomaly of the Bouguer slab.

In practical units: (density in gm/cm^3 and t in meters)

$$\Delta g_z = 0.04192 \, G \, \Delta\rho \, t \, \{ 1/2 + 1/\pi \tan^{-1}(x/h)\}$$

From this formula the $\Delta\rho t$ product is simply $\Delta g_{z\ max}/0.04192$. An independent estimate of the depth can be obtained from the horizontal derivative of the anomaly at x = 0 (the inflection point of the anomaly) via:

$$h = (\Delta g_{z\ max})/\pi \, (dg/dx)^{-1}$$

The horizontal cylinder of arbitrary cross section

The gravity anomaly of a horizontal cylinder whose cross section is an n-sided polygon was derived by Talwani et al. (1959). The code used to generate the anomaly in Figure 2.4.3 is based on Talwani's with modifications by Won and Bevis (1987). The polygon is a versatile way to represent any 2D cross section and the code is fast and robust.

Fundamentally the approach is to integrate the contributions of thin horizontal slabs over the cross section. The sketch below shows the form of this integration for one side of the polygon. In detail the integration is over the angles, θ, subtended by a side and the observation point. The principle of superposition is used to subtract the contribution of a second slab (shaded portion in the sketch) to leave the anomaly of the trapezoid. This procedure can readily be adapted to an n-sided polygon.

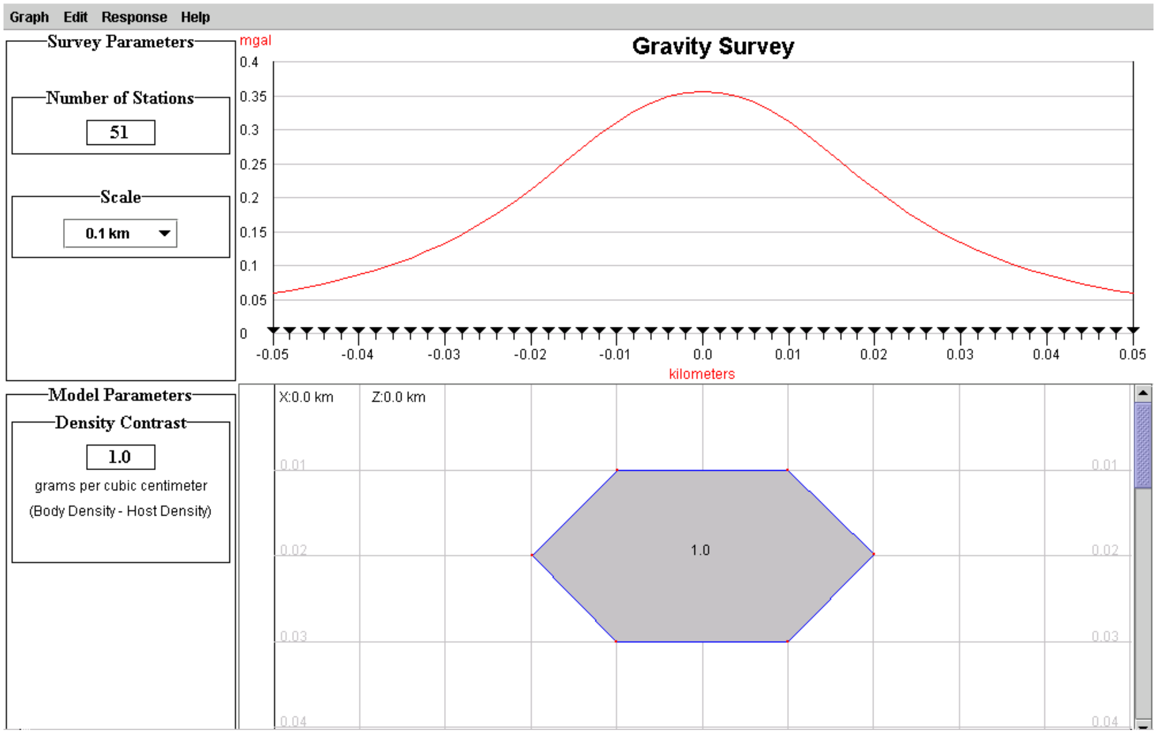

Graph Edit Response Help
Survey Parameters
Number of Stations
51
Scale
0.1 km
mgal
Gravity Survey
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.04
-0.03
-0.02
-0.01
0.0
0.01
0.02
0.03
0.04
0.05
kilometers
Model Parameters
Density Contrast
1.0
grams per cubic centimeter
(Body Density - Host Density)
X:0.0 km
Z:0.0 km
0.01
0.02
0.03
0.04
1.0

Figure 2.4.3