7.2.3 Time – Distance Plots

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In surface seismic surveys the ‘point’ source is located on the surface and detectors of the resulting seismic waves are located on the surface. The data of a survey are the arrival times of the wave fronts at various distances from the source. We have already seen a sample of this in the seismic time-traces that would be observed at geophones placed in a well adjacent to a surface source. The data are usually plotted with the arrival time on the vertical axis and the separation on the horizontal axis. The following cartoon shows a hypothetical surface reflection survey in which an array of 8 geophones is placed along a line on the surface at equal intervals from the source, S (usually called the *shot point*). The geophones in this example are located at 40, 80, 120, to 320 m, the layer is 200 m thick and the velocity is 500m/s.
The plot of wavelet arrivals vs. time at any particular geophone location is a recording of the ground motion at that geophone. This is the data recorded in a seismic survey and it is usually called the seismogram. The above schematic result is consequently known as a synthetic seismogram. The actual seismogram is considerably more complex because it displays the ground roll, refractions if there are any, and shear wave arrivals from part of the incident wave energy that is converted to shear energy at the interface.

The travel time curves for models with layered dipping interfaces can be calculated analytically and these formulas are well described in the standard texts referred to in the introduction to this chapter. These solutions should always be used to check any of the more general modeling codes.
The real world is rarely uniformly layered, certainly not with uniform layers of constant velocity. It is known that in sedimentary rocks the velocity increases with depth even in what appears to be a uniform depositional sequence. Further, velocities can vary laterally in a given geological unit because of depositional variation in grain size, clay content or degree of cementation. Finally the subsurface has structure. The goal of shallow surveys is often to map the depth to bedrock and this bedrock interface is unlikely to be a planar surface. Sedimentary layers have faults, anticlines, folds, and unconformities which are in fact the very features that trap petroleum and are the targets of the seismic exploration program in the first place.

The major task of modern exploration seismology is to develop models of the subsurface and methods of data processing which can be used to interpret the complex wave front arrivals on a typical seismogram. The numerical modeling programs that are used to create synthetic seismograms range from full 3 dimensional (3D) finite element or finite difference solutions to the governing wave equation to approximate solutions that trace the progress of particular rays through the medium. In this course we have adopted a general ray tracing program for creating travel time curves. This code will be used for modeling reflections and refractions from simple planar interfaces in the discussion that follows.

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**Reflection time-distance plots**

Consider a source (shot point) at point A with geophones spread out along the x-axis on either side of the shot point.
A raypath from A to C or A to E is: \[ 2 \sqrt{h^2 + \left(\frac{x}{2}\right)^2} \]

The travel time, \( t \), is the raypath divided by the velocity, \( V_1 \), or:

\[ t = \frac{\sqrt{x^2 + 4h^2}}{V_1} \]

Rearranging:

\[ \frac{V_1^2 t^2}{4h^2} - \frac{x^2}{4h^2} = 1 \]

This is the equation of a hyperbola symmetric about the \( t \) axis. The travel time plot for the direct wave arrivals and the reflected arrivals are shown in the following plot. The first layer is 100 m thick and its velocity is 500 m/s. The intercept of the reflected arrival on the \( t \) axis, \( t_i \), is the two-way zero
offset time and for this model is equal to 400ms. At large offsets the hyperbola asymptotes to the direct wave with slope $1/V_1$.

![Graph showing offset time and wave behavior](image)

In most seismic reflection surveys the geophones are placed at offsets small compared to the depth of the reflector. Under this condition an approximate expression can be derived via:

$$t^2 = \frac{4h^2}{V_1^2} + \frac{x^2}{V_1^2}$$

which can be rewritten as:

$$t = \frac{2h}{V_1} \left[ 1 + \left( \frac{x}{2h} \right)^2 \right]^{1/2}$$

or since $\frac{2h}{V_1} = t_i$ ,

$$t = t_i \left[ 1 + \left( \frac{x}{V_1 t_i} \right)^2 \right]^{1/2}$$
Since $\frac{x}{V_{1}t_{i}}$ is less than 1, the square root can be expanded with the binomial expansion. Keeping only the first term in the expansion the following expression for the travel time is obtained:

$$t = t_{i} \left[ 1 + \frac{1}{2} \left( \frac{x}{V_{1}t_{i}} \right)^{2} \right]$$

This is the basic travel time equation that is used as the starting point for the interpretation of most reflection surveys.

**Moveout**

A useful parameter for characterizing and interpreting reflection arrivals is the *moveout*, the difference in travel times to two offset distances. The following expanded plot of one side of the hyperbola of the previous reflection plot shows the moveout, $\Delta t$, for two small offsets.
Using the small offset travel time expression for $x_1$ and $x_2$ yields the following expression for the moveout:

$$\Delta t = \frac{x_2^2 - x_1^2}{2V_1^2 t_i}$$

The *normal moveout* (NMO), $\Delta t_n$, is a special term used for the moveout when $x_1$ is zero. The NMO for an offset $x$ is then:
The NMO is readily measured with small offset reflection data. With the value of the intercept time, $t_i$, the velocity is determined via:

$$V_1 = \frac{x}{(2t_i\Delta t_n)^{1/2}}$$

and the depth is then determined by:

$$h = \frac{V_1t_i}{2}$$

For a given offset the NMO decreases as the reflector depth increases and/or as the velocity increases.

In a layered medium the velocity obtained from the NMO of a deep reflector is an average of the intervening layer velocities. Dix (1955) found that the root-mean-square velocity defined by:

$$V_{rms} = \left( \frac{\sum_{i=1}^{n} V_i^2 t_i}{\sum_{i=1}^{n} t_i^2} \right)^{1/2}$$

where $V_i$ is the velocity in layer $i$ and $t_i$ is the travel time in layer $i$ is the best average to use.

In interpretation the NMO’s for successive reflections are used to obtain the average velocity to each reflector. Assuming these are the $V_{rms}$ velocities defined above then Dix (1955) showed that the velocity in the layer bounded by the $n^{th}$ and $n-1^{th}$ layer is given by:
Dip moveout

If the interface is dipping as in the figure below the up-dip and down-dip travel times are changed by an amount dependant on the dip angle $\theta$. The time-distance plot is still a hyperbola but the axis of symmetry is shifted up-dip by $2h \sin \theta$. (Shown by the dashed line in the figure. Note also that the depth is still the perpendicular distance from the interface to the shot point). The binomial expansion for the travel time for small offsets becomes:

$$t = t_i \left( 1 + \frac{x^2 + 4xh \sin \theta}{2V^2 t_i^2} \right)$$

For geophones offset a distance $x$ up-dip and down-dip, the dip moveout is defined as:

$$dip\ moveout = \Delta t_d = t_+ - t_ - x = \frac{2x \sin \theta}{V}$$

For small dips when $\sin \theta \approx \theta$, the dip moveout yields the dip via:

$$\theta \approx \frac{V \Delta t_d}{2x}$$

The velocity can be obtained with sufficient accuracy by averaging the velocities obtained in the usual manner from the up-dip and down-dip NMO’s.
Reflection survey configuration

There have been many configurations of shot point and geophone arrays used over the years. One important array is illustrated in the figure below. The geophone array, also called the spread, is laid out almost continuously along the profile. Shots are placed at the same locations as the geophones. At each shot point, $S_i$, recordings are made of the seismic record at each $n$ geophones on either side of $S_i$. After a succession of shots e.g. $S_{i+1}$, $S_{i+2}$, $S_{i+3}$, the geophone traces corresponding to rays that reflect at a common depth point (CDP) are collected and plotted.
This collection of records is also called a common midpoint array. The advantage of such an array is that many reflections from the same portion of the reflecting interface can be averaged.

The resulting gather of traces will of course show the typical moveout of the reflector but now all the rays reflect off the same point (the assumption is made that the layer has a very small dip otherwise the rays will not have a common reflection point). In practice there are variations in moveout caused by near surface variations in velocity so the moveouts of each trace will vary but because the reflection point is common and an average moveout can be calculated from which the velocity can be obtained.

The background ground motions at separate geophones are assumed to be random, as are the variations in near surface velocity. With these assumptions one method of averaging is to assume a velocity and shift each trace back to its zero offset value by its moveout. If the data were perfect all the reflection arrivals would line up horizontally and the traces could all be added together to form an average zero offset trace. By successively
changing the velocity until the maximum average reflection is obtained the optimum velocity is determined. In this average trace the reflection event would be well defined but the adjacent noise would average towards zero. Even with reflections with variable moveouts, the average will lead to something greater than the noise average so this process still leads to the selection of an optimum velocity. This process of shifting and averaging the pairs of traces is called a CDP gather.

The final averaged time trace is plotted directly beneath mid point of the pairs making up the gather. In practice, up to 64 common mid point shot-receiver pairs may be averaged with this single CDP trace. (The number of pairs averaged in this manner is referred to as the fold of the CDP gather.) The entire process is repeated to produce another CDP trace, one interval, $\Delta x$, farther along the spread.

For most reflection surveys the traces shown are CDP gathers.

**Geophone arrays and spacial filtering**

A major problem in reflection surveying is the presence of a large amplitude Rayleigh wave. Including the Rayleigh wave in a typical trace time plot for a deep reflection usually shows that the Rayleigh wave often arrives just in the time window of short offset reflections.
The Rayleigh wave can be minimized by considering the seismic arrivals at two geophones spaced at half the wavelength of the Rayleigh wave.

If the output of two such geophones is summed the Rayleigh wave will produce no output. The reflected wave on the other hand is coming up at near vertical incidence and will be doubled in the summed output. More geophones at the correct spacing will continue to augment the reflected arrival while effectively canceling the Rayleigh wave.

In practice each receiver location in a reflection survey consists of a group of geophones whose spacing is chosen to cancel the Rayleigh wave. A test survey is conducted first to determine the Rayleigh velocity and frequency from which the wavelength is determined from \( \lambda = \frac{V}{f} \).

**Migration**

For any reflection array the interpreted reflection point is plotted directly beneath the midpoint of the shot receiver separation. This is a plotting convention because information about the possible dip or reflector
geometry is not generally available in the simple offset data. The apparent vertical section is distorted by this means of plotting because the actual reflector point is plotted beneath the mid point.

Consider the following sloping step on a reflecting interface. Assume the velocity is known. The plots for the for the zero offset reflection (e.g. The CDP gather) are plotted on the model. On the left and right of the sloping section the reflection section mimics the actual section. However for reflections such as ABA, which occur from point B on the slope the plotted point is at $B'$ – displaced to the right of the actual reflection point. Its ‘depth’ is just where the arc of radius AB intersects the vertical beneath A. The net effect is that the plot of apparent reflection points shifts the interface to the right and changes its slope.

![Diagram of sloping step on reflecting interface]

This description also suggests the means to correct the section. Whenever a sloping interface is found in the zero offset section the points on this interface are moved back along an arc centered at the shot point. The line tangent to all the arcs is the true position of the interface. The process
of shifting an apparent slope back to its true position in space is known as migration.

**Refraction time-distance plots**

A typical ray path for an incident ray refracted at the critical angle is made up of the lines ABDE shown in the figure below. The incident ray at the critical angle, AB, yields a reflection BC and generates the head wave which propagates along the interface. The wave front of the head wave generates waves which return to the surface along rays which leave the interface at the critical angle, e.g path DE in the figure. The refraction arrivals consequently begin at the same time as the reflected wave on path ABC. Subsequent refraction arrivals are delayed by their travel time along the interface at the velocity of the lower medium.
The equation for the travel time to an arbitrary point on the surface is the sum of the travel times along AB, BD, and DE. The first and third times are identical so:

\[ t = t_{AB} + t_{BD} + t_{DE} (= t_{AB}) \]

\[ t = \frac{2AB}{V_1} + \frac{BD}{V_2} \]
Using the geometry imposed by Snell’s Law this becomes:

\[ t = \frac{2h}{V_1 \cos \theta_c} + \frac{x - 2h \tan \theta_c}{V_2} \]

Since \( \theta_c \) is determined via the velocities, \( \sin \theta_c = \frac{V_1}{V_2} \), then the equation can be rewritten in terms of velocity as: (note \( \cos \theta_c = \frac{\sqrt{V_2^2 - V_1^2}}{V_1} \) and \( \tan \theta_c = \frac{V_1}{\sqrt{V_2^2 - V_1^2}} \))

\[ t = \frac{x}{V_2} + \frac{2h \sqrt{V_2^2 - V_1^2}}{V_1 V_2} \]

This is the equation of a straight line with slope \( 1/V_2 \) and an intercept on the \( t \) axis, \( t_i = \frac{2h \sqrt{V_2^2 - V_1^2}}{V_1 V_2} \). This is the mathematical intercept; there are no refracted arrivals at distances less than AC or at times less than the reflection travel time for the ABC path.

The velocities can be determined directly from the travel time plot as the inverse of the slopes of the direct and refracted arrivals so the depth can be determined from the intercept time via:

\[ h = \frac{t_i V_1 V_2}{2 \sqrt{V_2^2 - V_1^2}} \]

The distance AC at which the first refraction arrives, called the critical distance, \( x_c \), can be obtained from:
\[
\frac{x_c}{2h} = \tan \theta_c = \frac{V_1}{\sqrt{V_2^2 - V_1^2}}
\]

so
\[
x_c = \frac{2hV_1}{\sqrt{V_2^2 - V_1^2}}
\]

Finally it can be seen from the time-distance plot that there is a distance after which the refracted arrivals come before the direct arrivals. This occurs at the crossover distance, \(x_{cross}\), when the refraction and direct waves have equal travel times, i.e when
\[
\frac{x_{cross}}{V_1} = \frac{x_{cross}}{V_2} + \frac{2hV_1}{V_2}\left(\frac{V_2^2 - V_1^2}{V_1V_2}\right)
\]

or when
\[
x_{cross} = 2h\left(\frac{V_2 + V_1}{V_2 - V_1}\right)^{\frac{1}{2}}
\]

This is another useful equation for determining \(h\). In practice with real data it is usually found that projecting the refracted arrivals back to the \(t\) axis to find the intercept time is more accurate than estimating where the crossover distance is.

The refraction arrivals from shot points at each end of a survey line over a dipping interface are shown in the following figure:
The arrivals at geophones down dip from shot point A come at progressively later times than their horizontal interface counterparts so that the slope of the arrival curve is steeper. The apparent velocity obtained from the plot, $V_{app \, down \, dip}$, is less than $V_2$. The apparent up dip velocity obtained with geophones up dip from shot point B is greater than $V_2$. The travel times from A to B and from B to A, the reciprocal times, must be the same. Refraction surveys must be shot in both directions. Arrival times taken in only one direction and interpreted as being taken over a horizontal interface may yield erroneous results if the interface is dipping.

The equations for the travel times for a dipping interface, and for multiple layers with dipping or horizontal interfaces, are derived analytically.
in Telford et al. (1990) and they present a useful collection of expressions for finding the depths and dips for up to three layer models. A particularly useful result for small dips is that

\[ \frac{1}{V_2} \approx \frac{1}{2} \left( \frac{1}{V_d} + \frac{1}{V_u} \right) \]

where \( V_d \) and \( V_u \) are abbreviations for the down dip and up dip apparent velocities respectively.

General expressions have been derived for the travel times for any number of layers with accompanying equations for depths and true velocities but the quality of the field time-distance data makes it difficult to identify intercept times or cross over distances for more than a few refraction arrival segments. A better approach which leads into general methods of interpreting seismic data is to use a numerical technique to generate arrivals in model of an arbitrary medium and then by a process known as inversion adjust the parameters of the model to match the observed data.

In summary the principal advantage of the refraction method over the reflection method is that it depends only on measuring the first arrival times on a seismic time trace. There is no problem separating the refracted arrival from other arrivals as there is in picking reflection events. Problems or disadvantages are:

i) there is no evidence in the travel time plot for an intermediate layer(s) of lower velocity than the layers enclosing it. Interpretation in this case, which assumes a progressive increase in layer velocity with depth, will be in error.

ii) there are situations where, even with increasing velocity in successive layers, a refraction arrival segment may be masked by a deeper higher velocity earlier arriving segment.
iii) the surface distribution of geophones must extend to distances of several times the anticipated depth of the refractor in order to identify the crossover distance and to determine the slope of the refractor arrival plot.

iv) at the large off-sets required by iii) the arrivals may be very weak and impractically big shot energies may be required

**The ray-tracing algorithm**

A numerical ray-tracing algorithm has been developed by Dr. John Washbourne for calculating the arrival times of P waves at any offset distances over a model consisting of three layers with interfaces of arbitrary dip. Both reflections and refractions are modeled. The algorithm itself can handle interfaces of arbitrary shape (anticlines, synclines, fault off-sets etc.) but for this course the interfaces are constrained to be planar. The ray tracing method, as the name implies, simply sums the time delays for incremental distances along a large number of rays that start out in all directions from the source. Each starting ray is tracked, summing the times as it inches along its path. At an interface a reflected ray is launched which is tracked along its path as is a refracted wave which is tracked along its path. Depending on the angular coverage of the initial rays a particular ray path may come out at a desired geophone location. If not interpolation of arrival times at points surrounding the desired point is used. The process sounds cumbersome and tedious but on any current computer the travel times are computed in the blink of an eye.

The interface for the Java applet for this algorithm is shown below. The left panel inputs the parameters of the survey (shot point location within
the array (Source X), first geophone location (First Rec X), geophone spacing (Recr X Incr) and number of geophones in the array), layered model (depth of layer one (Layer 1 Z) and layer two (Layer 2 Z) beneath the leftmost point of the array, depth increment of the layer to a point beneath the rightmost point of the array (Layer 1 DZ and Layer 2 DZ), and the layer velocities. After inputting these parameters press Calculate to generate the results. Alternatively the default layers can be dragged into new positions by moving the start and end points of the layer boundaries with the cursor. The shot point can similarly be moved within the array by dragging with the cursor. These latter two cursor operations result in near simultaneous recalculation of the travel time plots shown in the time-distance plot in the bottom panel.

The arrivals to be displayed can be selected from the menu presented by clicking Arrivals on the top menu bar. The default presentation shows all the arrivals.

Field data can be plotted by clicking on Field Data on the top menu bar and entering the measured times and offsets for the observed events. Event 1 might be the direct arrivals, Event 2 might be the refracted arrivals, Event 3 the suspected reflection arrivals, etc. The field data points are plotted with their event number as the plotting symbol. The default model includes a sample set of data for Event 1.

Finally an expanded graph of the time-distance plots can be seen by clicking on View Seismic Plot Only on the second row menu bar. This plot is normally used in detail analysis and interpretation. In both time-distance plots the cursor position in time and distance is presented in a coordinate box above the plot.