5.4 Finite conductors in a conducting half space

The general solution for the response of a conducting object buried in a conducting, and usually inhomogeneous, ground is a difficult problem in numerical analysis. A schematic of a system employing a horizontal loop transmitter (vertical dipole moment, M_Z) on the surface of the ground and adjacent to a spherical target is presented in Figure 5.9. We have seen the nature of the response of the target alone and of the half-space alone: it is the coupling between the two that poses the difficulty. There are three types of response from the target:

1) If the target has a magnetic permeability a zero frequency (DC) inducing field will magnetize it as we have seen in Figure 5.3. The resulting DC magnetic moment, M_{DC} , and its secondary magnetic field (sketched in dashed black lines in Figure 5.9) are **in the direction of the primary field** at the target. The primary field may actually be time varying as long as the time rate of change is so small that there is negligible Faraday induction.

2) A changing primary field will in general induce circulating currents, shown as closed green loops in Figure 5.9. The orientation and direction of the currents is such as to produce an induced magnetic moment, M_{AC}, and associated secondary magnetic fields sketched in red in the figure, which are in **the direction opposite to the primary field** at the target. The induced dipole moment is proportional to the induction number of the target which is the product $\sqrt{\omega\sigma\mu}\cdot r$ where r is some characteristic dimension of the target. In the frequency domain the net secondary field at a receiver changes in amplitude and phase and reverses direction as the frequency of the inducing field increases from DC. In TDEM systems there is a DC value for the secondary field during the on-time of the primary field and the field then decays with a time constant proportional to the product $\sigma\mu r^2$.

3) The transmitter induces currents in the half-space which circulate horizontally and concentrically with the transmitter loop (green lines in Figure 5.9b). The half-space currents are gathered or channeled into the conducting target as sketched in Figure 5.9b. The excess current in the target forms an induced electric dipole moment P (green) which produces secondary magnetic fields that circulate about P as sketched in Figure 5.9c.

An active EM system measures secondary fields from the half-space, from the DC magnetization, the AC induction magnetization and from the electric dipole moment caused by currents channeled into the target from the half-space. In principle all these effects are coupled electromagnetically and a rigorous mathematical solution is a challenging numerical modeling problem. For certain situations when the induction numbers for the target and enclosing half-space are different and for low induction numbers of the half-space, the various effects are approximately separable. This is the case for detecting and identifying relatively small metallic objects near the surface.

The secondary fields depend strongly on the size and shape of the target and it is this dependence that allows characterization, and possibly identification, of the target as well as its location. The dependence on shape is sketched in the cartoon of Figure 5.10. A spherical shell, a short cylindrical shell and a flat disk are used to represent a range of targets. For each we assume that the primary field can be oriented arbitrarily with respect to the coordinate axes. To a first approximation the induced moments can be represented by the simple dipole moments indicated by the black arrows, DC magnetization, and red arrows, AC induction.

These simple shapes show how diagnostic the AC and DC moments are for determining the shape and orientation of an object. It can be seen that multiple polarizations of the inducing field are required. For example, the currents induced in a cylindrical metal shell when the inducing field is oriented in the direction of its axis are different from the currents induced when the field is perpendicular to its axis. On the other hand, the currents induced in a sphere are always equal, regardless of the orientation of the field with respect to the sphere. For the disk the DC moments are only radial whereas the induced AC moment is only normal. By using multiple transmitters fields can be induced in three separate directions stimulating the possible moments in the object.