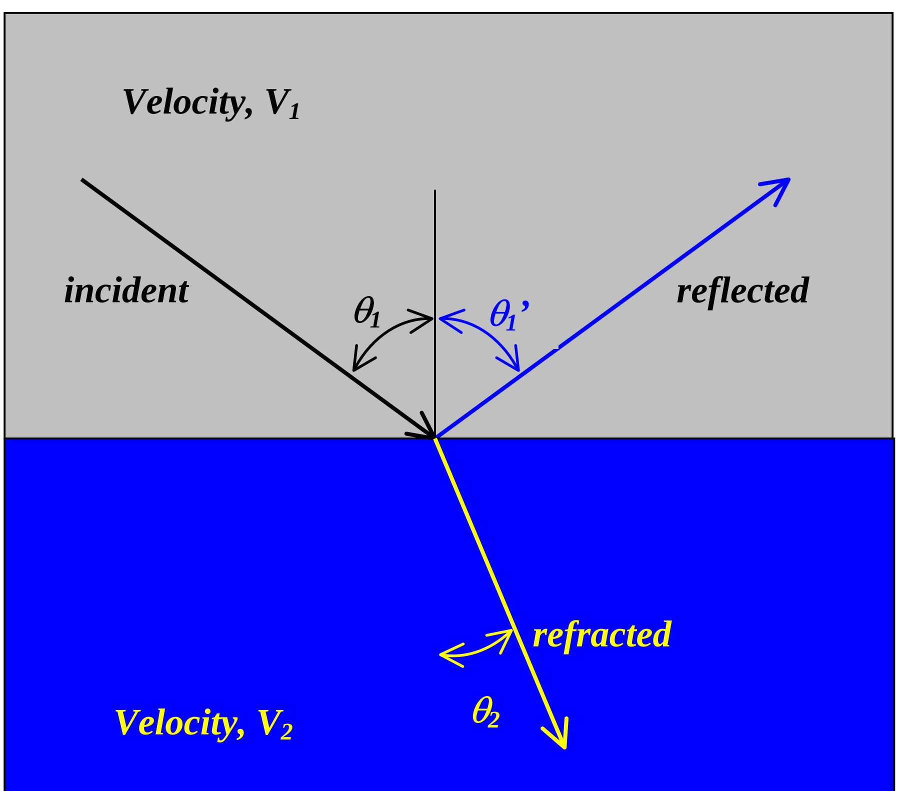
### 7.2.2 Reflection and Refraction



### The geometry of reflection and refraction

A wave incident on a boundary separating two media is reflected back into the first medium and some of the energy is transmitted, or refracted, into the second. The geometry of refraction and reflection is governed by Snell's Law which relates the angles of incidence, reflection and refraction to the velocities of the medium.

The cartoon below illustrates the ray geometry for a P-wave incident on the boundary between media of velocity $V_1$ and $V_2$. The angles of incidence, reflection and refraction, $\theta_1, \theta_1'$, and $\theta_2$, respectively are the angles the ray makes with the normal to the interface.



Snell's Law states that: $$\frac{Sin\theta_1}{V_1} = \frac{Sin\theta_1'}{V_1} = \frac{Sin\theta_2}{V_2}$$

Snell's law requires that the angle of reflection is equal to the angle of incidence. It further implies that if $V_2$ is less than $V_1$ the ray is bent towards the normal to the interface as in the cartoon, but if $V_2$ is greater than $V_1$ the ray is bent away from the normal.

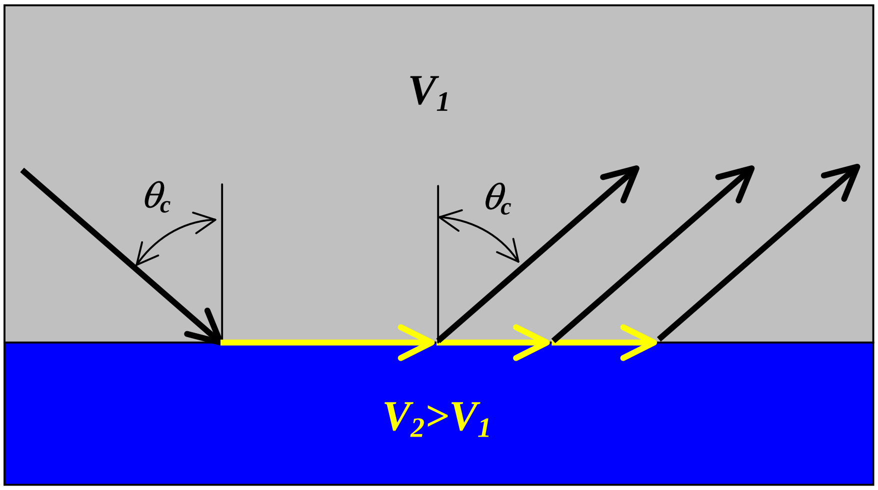
It is instructive to observe the progress of a spherically spreading wave as it impinges on a horizontal boundary between media of different velocity. The wave fronts are definitely not planar nevertheless rays at the interface satisfy Snell's Law. As in the planar model used to derive Snell's Law, each point on the interface where the spreading wave hits becomes the source of new waves which propagate back into medium one and on into medium 2. In the following movie the spreading wave is reflected at the interface and refracted into a medium of lower velocity.

[*Click here to play the movie*]

If $V_2$ is greater than $V_1$ the angle of refraction is greater than the angle of incidence. The latter result can lead to a special condition where $\theta_2 = 90°$ and $Sin\theta_2 = 1$. The angle of incidence for which this occurs is called the *critical angle*, $\theta_c$. The critical angle is given by;

$$\theta_c = Sin^{-1}\left(\frac{V_1}{V_2}\right)$$

The geometry of this wave is correctly given by Snell's law but the nature of a wave that propagates along the interface is not so simple. The critically refracted wave is a disturbance that propagates along the interface with the velocity of the lower medium. As it goes its wave front acts as a moving source of waves that propagate back into the upper medium with velocity $V_1$ along rays which are parallel to the reflected wave at the critical angle. This phenomenon is illustrated in the following cartoon.




The following movie shows the propagation of an expanding wav front as it moves through such a boundary.

[*Click here to play the movie*]

The wave front in the lower, higher velocity, layer moves faster than the incident wave front along the boundary. In the plane of the boundary this creates a stress disturbance which travels along the boundary with velocity $V_2$ and at each point acts as a source for waves propagating back up to the surface. This wave is commonly known as a Head wave and its wave front is clearly seen beyond the point on the interface where the incident ray is at the critical angle.

## Wave conversion and reflection coefficient

In the description of reflection and refraction up till now we have not discussed the physics of why a seismic wave is reflected only what the geometric relationship of the wavefronts must be as the wave crosses an interface. The energy that is reflected is determined by using the form for the solution for the particle displacement (e.g. the form $A_0 e^{i(\omega t - kx)}$ that we saw for a wave traveling in the x direction) for a wave traveling towards an interface (the incident wave), one traveling away from the interface in the opposite direction to the incident wave (the reflected wave) and finally one transmitted into the second medium. At the interface these general solutions must satisfy the boundary conditions: the displacement across the interface must be continuous and both the normal and tangential stress must be continuous.

For a P wave incident at an arbitrary angle these boundary conditions cannot be met without incorporating an $S_V$ wave as a reflected and transmitted wave. Thus it appears that the incident P wave generates an $S_V$ wave at the interface. [Note that the $S_V$ waves leave the interface at angles dictated by Snell's Law: $\frac{\sin\theta_{1P}}{V_{1P}} = \frac{\sin\theta_{2P}}{V_{2P}} = \frac{\sin\theta_{1S}}{V_{1S}} = \frac{\sin\theta_{1S}}{V_{2S}}$ ].

The solutions for the amplitudes of the various P and $S_V$ waves for a P wave incident at an arbitrary angle are given by the Zoeppritz Equations. In general they yield complex results that cannot be summarized in this introductory treatment. The analysis of the amplitude of the reflected waves as a function of incident angle, which in practice means as a function of shot-point geophone off-set, yields valuable information about the rock properties on either side of the interface. This forms the basis of the important amplitude vs. offset (AVO) analysis of modern reflection seismology.

The reflection coefficient for small off-sets is very close to that for normal incidence, and since for normal incidence the stresses tangent to the interface are zero, the dependence on the shear component disappears and no shear wave is generated. The reflection coefficient from the Zoeppritz equations takes on a very simple form:

$$\frac{\textbf{\textit{Amplitude of reflected wave}}}{\textbf{\textit{Amplitude of incident wave}}} = \boldsymbol{R} = \frac{\rho_2 \boldsymbol{V}_2 - \rho_1 \boldsymbol{V}_1}{\rho_2 \boldsymbol{V}_2 + \rho_1 \boldsymbol{V}_1}$$

Similarly the transmission coefficient is:

$$\frac{\textbf{\textit{Amplitude of transmitted wave}}}{\textbf{\textit{Amplitude of incident wave}}} = \boldsymbol{T} = \frac{2\rho_1 \boldsymbol{V}_1}{\rho_2 \boldsymbol{V}_2 + \rho_1 \boldsymbol{V}_1}$$

Note that it is the energy that is conserved in reflection and transmission and the energy is proportional to the square of the amplitudes of the waves. The energy reflection coefficients are the squares of the above amplitude reflection coefficients. This is important because it will be noted that the sum of the fraction of the amplitude reflected and the fraction transmitted do not generally add up to one whereas the energy coefficients do.